Written Exam for the M.Sc. in Economics Autumn 2014 (Fall Term)

Financial Econometrics A: Volatility Modelling

Final Exam: Masters course

Exam date: **February 20 2015**

3-hour closed book exam.

Please note there are a total of 9 questions which should **all be replied to**.

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Question 1:

Q1.1:

Consider the t_3 ARCH(1) model for the returns x_t as given by,

$$x_t = \sigma_t z_t, \tag{1.1}$$

where the z_t are i.i.d.$t_3(0,1)$ distributed. That is, z_t is scaled t distributed with 3 degrees of freedom such that, in particular, $Ez_t = 0$ and $Ez_t^2 = 1$. Moreover, the conditional variance is given by,

$$\sigma_t^2 = \omega + \alpha x_{t-1}^2, \quad \omega > 0, \quad \alpha \geq 0. \tag{1.2}$$

Is $E|z_t|^3$ finite?

Explain why this model would potentially be more interesting for analysis of stock returns when compared to the case where z_t are i.i.d.N$(0,1)$ distributed.

Q1.2:

Use the drift function $\delta(x) = 1 + x^2$ to derive a condition for x_t to be weakly mixing with finite variance.

Q1.3:

The log-likelihood contribution $l_t(\theta)$ at time t for the ARCH in terms of $\theta = (\omega, \alpha)$ is given by (up to constants)

$$l_t(\theta) = -\log \sigma_t^2 - 4 \log\left(1 + \frac{x_t^2}{\sigma_t^2}\right).$$

It follows that (do not show this) that the score at θ_0 is given by

$$\tfrac{1}{\sqrt{T}} S_T(\theta_0) = \tfrac{1}{\sqrt{T}} \sum_{t=1}^{T} \frac{\partial l_t(\theta_0)}{\partial \alpha} = \tfrac{1}{\sqrt{T}} \sum_{t=1}^{T} \left[\frac{3z_t^2 - 1}{1 + z_t^2}\right] \left(\frac{x_{t-1}^2}{\omega_0 + \alpha_0 x_{t-1}^2}\right),$$

where it is used that by definition $z_t = x_t/\sigma_t$.

It can be shown that the series $y_t := \left[\frac{3z_t^2-1}{1+z_t^2}\right]$ is i.i.d. with $Ey_t = 0$, $Ey_t^2 < \infty$ and $Ey_t^4 < \infty$.

Use this to argue that $\frac{1}{\sqrt{T}} S_T(\theta_0)$ is asympotically Gaussian distributed for $\omega_0 = 1$ and $\alpha_0 = 0.5$.

Q1.4:

The result in Q1.3 is one of the regularity conditions to establish that for the MLE $\hat{\alpha}$ of α it holds that $\sqrt{T}\,(\hat{\alpha} - \alpha_0)$ is asymptotically Gaussian.

List the missing regularity conditions which imply this.

Q1.5:

Explain how you would use the model in (1.1)-(1.2) to compute 5% conditional VaR for x_t.

Question 2:

Q2.1:

Consider the log-price series y_t in Figure 2.1 with t=1,2,...,T=1620.

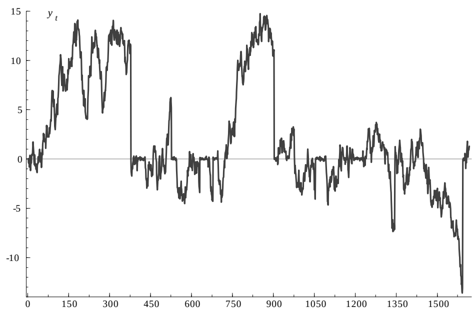

Figure 2.1

For estimation the following 2-state switching model was applied:

$$\Delta y_t = \sigma_{s_t} z_t, \quad t = 1, 2, ..., T. \tag{2.1}$$

Here z_t are i.i.d.N(0,1) distributed, and the switching variable $s_t \in \{1, 2\}$, with the switching governed by the transition matrix $P = (p_{ij})_{i,j=1,2}$. Moreover,

$$\sigma^2_{s_t} = \sigma^2_1 1\,(s_t = 1) + \sigma^2_2 1\,(s_t = 2)\,. \tag{2.2}$$

Gaussian likelihood estimation gave the following output, with misspecification tests in terms of smoothed standardized residuals $\hat{z}^*_t$:

$\hat{p}_{11} = 0.98$ $\quad$ $\hat{p}_{21} = 0.07$	$\hat{\sigma}^2_1 = 0.64$ and $\hat{\sigma}^2_2 = 0.2$
LM-test for Normality of $\hat{z}^*_t$:	p-value: 0.12
LM-test for no ARCH in $\hat{z}^*_t$:	p-value: 0.10

Interpret the model. What would you conclude on the basis of the output and Figure 2.1?

Q2.2:

In order to find the MLE of $\theta = (p_{11}, p_{21}, \sigma_1^2, \sigma_2^2)$ the function $M(\theta)$ given by,

$$M(\theta) = \sum_{i,j=1}^{2} \log p_{ij} \sum_{t=2}^{T} p_t^*(i,j) + \sum_{j=1}^{2} \sum_{t=2}^{T} p_t^*(j) \log f_\theta(\Delta y_t | j), \qquad (2.3)$$

can be used.

Provide an expression for $f_\theta(\Delta y_t | 1)$.

Explain what $p_t^*(i,j)$ are.

What would change if z_t where assumed i.i.d.$t_v(0,1)$ distributed for some $v > 2$?

Q2.3:

Explain how you would use $M(\theta)$ from (2.3) in order to find the MLE $\hat{\theta}$.

Comment on what Figure 2.2 shows in relation to finding $\hat{\theta}$.

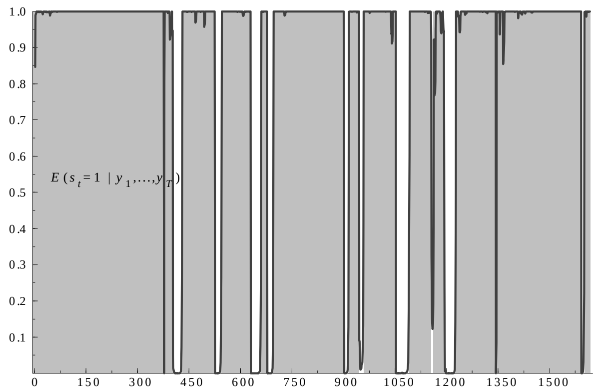

Figure 2.2

Q2.4:

Now assume that at time T, $s_T = 2$. In order to forecast if one will enter state 1 at $T+1$, use the out-put to provide an estimate of $P(s_{T+1} = 1 | s_T = 2)$.

Derive or give an expression for $P(s_{T+2} = 1 | s_T = 2)$. Provide an estimate for this based on the output.